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# Company valuation as result of risk analysis: replication approach as an alternative to the CAPM

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Market imperfections call into question the suitability of the CAPM for deriving the cost of capital. The valuation by incomplete replication introduces a valuation concept that takes capital market imperfections into account and derives the risk-adjusted cost of capital (or risk discounts) on the basis of corporate or investment planning and risk analysis. The risk measure is derived consistently (using risk analysis and Monte Carlo simulation) from the cash flows to be valued, that is, the earning risk. Historical stock returns of the valuation object are therefore not necessary. It can be shown that the valuation result of the CAPM can be derived using the approach of imperfect replication as a special case for perfect capital markets.

## 1. Introduction and overview

The idea of a capital market-oriented<sup>1</sup> company valuation has to be questioned due to many imperfections<sup>2</sup> of the capital market<sup>3</sup>. Especially the CAPM does not meet the challenges of a company valuation on imperfect capital markets due to the assumption of perfect and complete capital markets<sup>4</sup>. An improvement in the valuation results (e.g. as a basis for decision-making on the purchase of companies) if company-related factors (such as growth<sup>5</sup>, return on equity or company-specific risks) are taken into account in the valuation models<sup>6</sup>. This leads to a replacement of capital market-oriented valuation approaches with procedures that - in the tradition of investment theoretical valuation approaches<sup>7</sup> - deal with earnings risks and not primarily with share price fluctuations; these are the semi-investment theoretical valuation methods described in this article, which are based on an analysis of business risks and the method of imperfect replication. The central advantage of the valuation approach is that it is based on only two low restrictive assumptions and, in particular, does not make any restrictive assumptions about the characteristics of the valuation subject or the capital market. In particular, there is no

need to assume a perfect or complete or arbitrage-free capital market (the central assumption is simply the following: two payments at the same time have the same value if they match the expected value and the risk measure chosen by the valuation subject). Also rating and financing restrictions and insolvency costs are possible. Overall, an “idealized market calculus”, as explained by *Ballwieser* (2010), is therefore not required for deriving the valuation equations<sup>8</sup>. So the actual approach presents an alternative to CAPM and implied cost of capital (*Bini*, 2018) to derive cost of capital. But it is not necessary to assume that the value is the market price. The valuation always takes place consistently from the perspective of the respective valuation subject (so that, for example, the degree of diversification of its assets achieved by this valuation subject always prevails, and not the diversification possibilities of other valuation subjects on the capital market).

In contrast to pure investment theoretical methods<sup>9</sup>, the valuation (and the derivation of the cost of capital) takes place without the necessity of considering and simultaneously optimizing all investment and financing possibilities of the valuation subject (espe-

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<sup>1</sup> I.e. financial theoretical.

<sup>2</sup> See the overview of empirical studies at *Gleißner*, 2014.

<sup>3</sup> *Hering* 2014.

<sup>4</sup> See *Bini*, 2018, pp. 8-11; *Dempsey* 2013a and b; *Fernandez* 2013 and 2017; *Rossi*, 2016; the empirical studies (and alternative models) at *Fama/French*, 2015; *Blitz/Hanauer/Vidojevic/van Vliet*, 2018; *Kaserer/Hanauer*, 2017; *Ang et al* 2006 and 2009; *De Bondt/Thaler*, 1985 and 1987;

*Jegadeesh/Titman*, 2011.

<sup>5</sup> Esp. of assets (see *Chen/Novy-Marx/Zhang*, 2011 and *Fama/French*, 2015).

<sup>6</sup> See *Ang et al* 2006 and 2009; the empirical studies (and alternative models) at *Fama/French*, 2015; *Kaserer/Hanauer*, 2017; *Walkshäusl*, 2013; *Zhang*, 2009.

<sup>7</sup> See *Matschke/Brösel*, 2013 and *Hering*, 2014.

<sup>8</sup> The explained semi-investment-theoretical valuation approach is new due to the lack of need for an idealized capital market or the existence of a utility function. It is not included in the survey of *Ballwieser*, 2010.

<sup>9</sup> See i.e. *Hering*, 2014; *Matschke/Brösel*, 2013; *Toll/Kintzel*, 2018.

cially by means of a simplex algorithm)<sup>10</sup>. In the tradition of risk-value models<sup>11</sup>, valuation is performed by comparing the expected value of cash flows and their risks, expressed by a selected risk measure, with the risk-return profile of alternative investment opportunities (e.g. government bonds and equity indices available on the capital market). In accordance with the idea of “imperfect replication”, the risky cash flow to be valued is thus expressed only in terms of the expected value and risk measure (R). It is only necessary to know the relevant information about two alternative investment opportunities (and not about the whole investment program)<sup>12</sup>. Accordingly, the valuation is based on a  $(\mu, R)$ -preference function that includes the well-known  $(\mu, s)$ -preference function of the Capital Asset Pricing Model (CAPM) as a special case. In contrast to the utility theoretical evaluation, knowledge of utility functions is also not required<sup>13</sup>. The great advantage of the valuation approach is that no (historical) capital market information about a company to be valued is required and the derivation of the cost of capital and company value from the analysis of the opportunities and risks of the company is possible. Risk analysis and Monte Carlo simulation for the aggregation of individual risks with reference to corporate planning provide the valuation-relevant information. Due to the consistent reference to the future and the consideration of future risks, the valuation approach outlined in this article is suitable for valuing existing options for action in the preparation of business decisions (e.g. in the context of a strategy assessment). This also explains the great importance of the valuation approaches presented here for financial corporate management (controlling). The central business task is a well-founded weighing of expected returns and risks in important decisions. The preparation of business decisions requires a well-founded strategy, operational planning based on it, an analysis of opportunities and threats and a risk-adequate evaluation of the options for action.

In this article we first discuss the challenges of a modern company valuation. We then analyse how a risk adjustment is made using the risk premium method and the certainty equivalence method. We then apply the certainty equivalence method to the CAPM. In the next section, we derive the valuation equation and the cost of capital using incomplete replication as

an alternative to CAPM. Insolvency risk and rating are also taken into account.

The article is structured as follows. Section 2 addresses some of the key challenges of adequately capturing risks in the valuation of companies, such as the fact that business risks generally affect (1) the expected value of cash flows and (2) the cost of capital. Chapter 3 discusses the two ways in which business risk is accounted for in the valuation: the calculation of risk-adjusted cost of capital or of certainty equivalents. Section 4 shows how to derive valuation equations and cost of capital without assuming a perfect capital market (as in the case of the CAPM). In particular, the above-mentioned method makes it possible to derive risk-appropriate cost of capital directly from the results of the analysis of the company’s risks. Special attention is also paid to the significance of the insolvency risk as well as the often existing rating and financing restrictions for the shareholder value (section 5). Section 6 explains the method by means of a simple case study before a short summary of the key statements.

## 2. Effects of risk on company value

When determining the value of a company as a future success value, it is necessary to observe certain equivalence principles<sup>14</sup>. It must be ensured that the “numerator” and the “denominator” of the valuation equation(s) are consistent with each other, especially with regard to risk assessment. This applies regardless of whether the risk adjustment of the cash flows is “in the denominator”<sup>15</sup> (in the case of the risk premium method) or “in the numerator” (in the case of the risk discount method)<sup>16</sup>.

It should be noted that risks potentially affect (1) the expected value of the cash flow  $E(CF)$  and (2) the cost of capital<sup>17</sup> at the same time. The effect of a risk R is simplistically shown in graph 1.

<sup>10</sup> The evaluation is understood as a comparison procedure and not as an optimization procedure.

<sup>11</sup> See Sarin/Weber, 1993.

<sup>12</sup> This is why the term “semi-investment theory approach” is also used here, see Gleißner, 2011.

<sup>13</sup> See Bamberg/Dorfleitner/Krapp, 2006 and the overview at Schosser/Grottko, 2013.

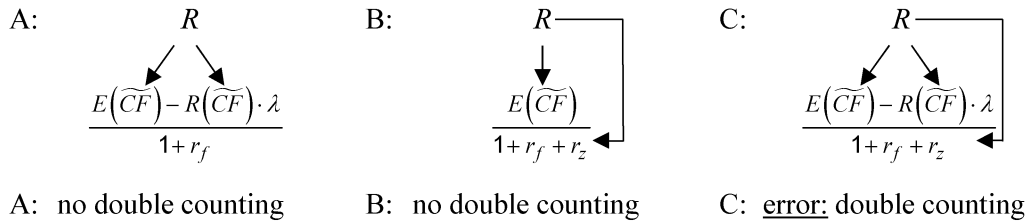
<sup>14</sup> See Moxter, 1983 and Dehmel/Hommel, 2017.

<sup>15</sup> Which is not recommended (see Spremann, 2004).

<sup>16</sup> See especially how to deal with insolvency risks that lead to a termination of the cash flow to the owner, Gleißner, 2017c.

<sup>17</sup> By means of a risk discount in the numerator or a risk premium  $r_z$  in the interest rate in the denominator.

**Graph 1: Impact of risk of a company on its valuation components**



This article deals with the methods of adequately recording risks in company valuation and shows in particular that the impact of risks on the expected value of cash flows (the numerator) and the discount rate (the denominator) can be derived consistently from a risk analysis of cash flows. An independent model for determining the discount rate - e.g. for deriving the CAPM beta based on fluctuations in equity returns or via a peer group - is not necessary<sup>18</sup>. The unrealistic assumptions of a perfect (or complete) capital market, as in financial theory valuation methods, are not required<sup>19</sup>.

In addition, risks also affect the probability of default (the rating) and, via it, the level of cost of debt and the development over time of the expected value of the cash flows (a special case of the expected value effects explained above, see section 5).

In the following, “semi-investment” theoretical valuation methods are presented that take capital market imperfections into account and consistently calculate risk-adjusted cost of capital (or risk discounts) on the basis of corporate or investment planning. The information from risk analysis, financing restrictions, and insolvency risks are taken into account. The procedures can also be used if no capital market data is available for non-listed companies because the valuation is consistently derived from the uncertain cash flows themselves (business plan).

### 3. Fundamentals of risk adjustment in the evaluation of series of cash flows

The company valuation is based on the discounted cash flow method (DCF). Under the DCF method, the value of a company is determined on the basis of expected future cash flows<sup>20</sup>. These expected cash flows are derived from an integrated planning calculation. To determine the company value, the cash flows are

discounted to the valuation date using a suitable capitalization interest rate (cost of capital).

The risk of future cash flows ( $\widetilde{CF}$ ), i.e. the extent of possible deviations from the expected value ( $E(\widetilde{CF})$ ) can be considered in the following two ways

- using the risk premium method, i.e. the calculation of the cost of capital<sup>21</sup>
- or using the certainty equivalence method (risk discount variant).

#### 3.1 Cost of capital: the risk premium method

With the risk premium method, a risk premium ( $r_{CF}$ ) is added to the risk-free interest rate ( $r_f$ ). This results in a discount rate ( $c$ ) (approximately the cost of capital) for discounting the expected future cash flows<sup>22</sup>. The formula for the discount rate is as follows:

$$(1) \quad c = r_f + r_{CF}$$

$r_{CF}$  is usually determined as a function of equity yield risks, e.g. expressed by the beta factor of the CAPM. The extent to which this reflects the actual risks of the company, e.g. the volatility of cash flows  $\sigma(\widetilde{CF})$  is, however, open. And only under specific additional assumptions, especially with regard to perfect capital market, the risks of the cash flows of the company are adequately recorded in  $r_{CF}$ .

The value of a risky cash flow ( $\widetilde{CF}_1$ ) at time  $t = 0$  is obtained by discounting the expected value  $E(\widetilde{CF}_1)$  with the cost of capital  $c$ :

$$(2) \quad Value(\widetilde{CF}_1) = \frac{E(\widetilde{CF}_1)}{1 + r_f + r_{CF}}$$

The risk premium method is often used in company valuation practice. However, it leads to valuation errors when a uniform risk premium is applied to both positive and negative cash flows<sup>23</sup>. This can be explained as follows. The basic idea behind discounting

<sup>18</sup> And generally not consistent with the valuation-relevant risk scope of the cash flow.

<sup>19</sup> See Ballwieser, 2008; see also the criticism at Dempsey, 2013; Gleißner, 2014; Hering, 2014 and Fernández, 2013 und 2017.

<sup>20</sup> I.e. free cash flows or flows to equity.

<sup>21</sup> This approach is usually applied in business valuation practice.

<sup>22</sup> But it is necessary to know the market price and to assume that the value is the price (see Black, 1986 and Shleifer/Vishny, 1992 und 1997 for some problems with this assumption).

<sup>23</sup> See Spremann, 2004, p. 253.

uncertain cash flows is that due to risk aversion, uncertain cash flows are assigned a lower value by discounting than certain cash flows. However, this is precisely not achieved by discounting negative cash flows: discounting negative cash flows increases the value because it becomes less negative<sup>24</sup>. It is therefore advisable to use the certainty equivalence method, which provides correct valuations<sup>25</sup>.

### 3.2 Certainty equivalence method

The certainty equivalence method is based on the following equation:

$$(3) \quad Value(\widetilde{CF}_1) = \frac{CE(\widetilde{CF}_1)}{1+r_f} = \frac{E(\widetilde{CF}_1) - \lambda_{CE} \cdot R(\widetilde{CF}_1)}{1+r_f}$$

$\lambda_{CE}$  stands for the “market price of risk”. This term expresses what additional return per unit for additionally accepted risk (measured in the selected risk measure  $R(\widetilde{CF})$ <sup>26</sup> for the alternative investment opportunity under consideration, e.g. the capital market) is to be expected. The scope of risk of a cash flow is recorded with a deduction in the numerator. A clear distinction is made between risk preference in the numerator and time preference (risk-free interest rate) in the denominator (Ballwieser, 1981).

The risk analysis of the cash flows to be valued leads to risk-adjusted risk measures that are not derived from historical stock returns. Suitable risk measures, such as value-at-risk, can take into account not only the standard deviation used in the beta factor but also the skewness and kurtosis of the distribution or a larger data density.

### 3.3 CAPM based on certainty equivalent and risk analysis as a special case

Even if an appraiser wishes to follow the traditional CAPM valuation approach, he or she should aggregate the valuation-relevant information on the risks of uncertain cash flows  $\widetilde{CF}$  to an appropriate risk measure. This is made possible by the “risk discount variant” of CAPM, whose risk measure is based on the correlation between future cash flows and the market return. The “risk discount variant” of the CAPM is also applicable if

- in the case of unlisted companies, there are no historical share price returns to calculate the beta factor, or
- historical returns cannot be regarded as representative for the future, for example, due to capital

market imperfections or a strategic decision, like change in the business model.

The risk discount variant or certainty equivalence variant of the CAPM is as follows:

$$(4) \quad Value(\widetilde{CF}_1) = \frac{E(\widetilde{CF}_1) - \rho(\widetilde{CF}_1, \widetilde{r}_M) \cdot \sigma(\widetilde{CF}_1) \cdot \frac{(r_m^e - r_f)}{\sigma(\widetilde{r}_m)}}{1+r_f}$$

with  $\rho$  as correlation coefficient of the uncertain cash flow and the uncertain return of the market  $\widetilde{r}_m$ ,  $\sigma(\widetilde{CF})$  as standard deviation of the expected cash flows (scope of risk expressed in monetary units) and  $r_m^e = E(\widetilde{r}_m)$  as expected return of the market portfolio (see Robichek/Myers, 1966; Rubinstein, 1973 and Gleißner/Wolfrum, 2009).

A future-oriented calculation of the correlation  $\rho$  is possible either through a so-called “risk factor approach”, which models joint influencing factors on  $\widetilde{CF}_1$  and the uncertain return of the market  $\widetilde{r}_M$  (e.g., economic situation, exchange rate, and oil price) or through a statistical analysis of historical data. It cannot be assumed that historical stock returns, which may also be influenced by psychological factors or momentum trading strategies, show the valuation-relevant risk of the cash flows to be valued (Dirrigl, 2009).

In contrast to the traditional CAPM return equation, the variant shown is also applicable to negative cash flows. For communication purposes, the valuation result can also be converted into a cost of capital rate (or an implicit beta factor).

The valuation equation for the risk discount variant of the CAPM can be derived using a robust replication approach even without the restrictive assumptions of the CAPM (see Gleißner/Wolfrum, 2009 and Dorfleitner/Gleißner, 2018).

## 4. Deriving the valuation equation and cost of capital from a risk analysis using incomplete replication

### 4.1 Deriving the valuation equation using incomplete replication

In the following, a so-called incomplete replication approach (“duplication”) is used to show how concrete valuation equations (and thus the market price of the risk  $\lambda_{CE}$  can be derived)<sup>27</sup>. Later in 4.2 we will derive the cost of capital.

It is of fundamental importance - and a key advantage - that the following valuation methodology, and the cost of capital derived later in section 4.2, are

<sup>24</sup> This is only correct in context of the market approach for a well-diversified shareholder and for cash flows with a negative correlation to the market returns.

<sup>25</sup> For derivation see Gleißner/Wolfrum, 2009.

<sup>26</sup> It is worth to mention that a risk measure  $R(\widetilde{CF})$  e.g.  $\sigma(\widetilde{CF})$  is not necessary if it is intended to get implied cost of capital (Bini, 2018).

<sup>27</sup> Gleißner, 2011 and Dorfleitner/Gleißner, 2018.

based on only a few, and less restrictive, assumptions that open up a broad field of application:

First assumption:

Two cash flows at the same time have the same value for the valuation subject if they match the expected value and the risk measure chosen by the valuation subject.

Second assumption:

For the subject of the valuation, a<sup>28</sup> risk-free investment with an interest rate  $r_f$  and a risk-bearing investment option with an uncertain return  $\tilde{r}_M$  (e.g. a broad empirical market portfolio) are available as alternative investment opportunities.

That's all. In particular, no further assumptions about the capital market are required (this does not have to be arbitrage-free or complete). Further assumptions about the subject of the valuation are required. It does not need to be perfectly rational nor perfectly diversified (due to ancillary assumptions, such as in CAPM).

In particular, no utility function of the evaluation subject must be known because its risk preference only manifests itself in the choice of the risk measure (which, incidentally, is similar in this respect to the CAPM), which specifically underlays a  $(\mu, \sigma)$  i.e. a special case of the here generally accepted  $(\mu, R)$

In order to determine the value of an uncertain cash flow  $CF_A$  of an investment  $A$  in a one-period model, an (incomplete) replication that is in line with expectations and is risk-adequate is carried out. Two investment options should be available for this purpose:

- the (empirical) market portfolio<sup>29</sup> with an uncertain return  $\tilde{r}_M$  and
- a risk-free investment with the interest rate  $r_f$ .

It is important to note that in contrast to CAPM or valuation methods based on the assumption of an arbitrage-free capital market<sup>30</sup> this valuation approach does not require any customary, restrictive (and less realistic) assumptions about the capital market. This is a significant advantage of the method explained here. With regard to the capital market, it is only assumed that there is a<sup>31</sup> risk-free investment opportunity and a risky investment opportunity (for example, the ability to invest in a broad market index, such as the MSCI All Country). In particular, it is not necessary to assume that the capital market is perfect, complete or arbitrage-free<sup>32</sup>. No assumptions are required regarding e.g. the absence of taxes or transaction costs. The risky investment opportunity, which can be understood as an

“empirical market portfolio”, need not have any other condition than that considered by the valuation subject as an investment opportunity<sup>33</sup>.

The market portfolio in this context is nothing more than a portfolio of uncertain assets that exist (and can be invested) in the real world.

It is a fundamental advantage of the methodology proposed here that it does not require any restrictive assumptions about (1) the capital market or (2) the behavior of the valuation subject (as explained above, the latter is not necessarily the - in reality non-existent – homo economicus, who, however, chooses operationalized optimal behavior, acts only according to the central assumption 1 above).

Unrealistic and restrictive assumptions are not required for deriving the valuation equations. In particular, the application of the valuation method also allows for constellations in which no sale of the company is envisaged at all (as discussed in the introduction situation of a strategy assessment).

In contrast, e.g. there are no assumptions for the CAPM that would imply that

- Value and price are basically the same,
- The valuation subjects would have perfectly diversified portfolios (and therefore would only bear systematic risks, as in the CAPM).

Value and price can differ so very well in the assumption system made here and valuation subjects - as in reality – are free to have diversified portfolios or not. Due to the lack of the need to use restrictive assumptions, it is possible in particular to cover existing constellations for the valuation which otherwise cannot be assessed (e.g. the evaluation of strategic options for action of an entrepreneur as a valuation subject who owns all his assets in his own company and thus carries company-specific risks).

It is easy to calculate the value  $(CF) = x + y$ . The amount of capital  $x$  invested in the market portfolio and the amount of capital  $y$  invested in the risk-free investment is exactly enough that the risk of this portfolio corresponds to the risk of the uncertain cash flow  $CF_A$ . The risk is measured by a suitable risk measure  $R(CF_A)$ , such as standard deviation, value-at-risk or conditional value-at-risk. The risk measure can generally be selected by the valuation subject and is an expression of the risk perception. In addition to the risk measure of the standard deviation which is usual in capital market-oriented valuation (especially the

<sup>28</sup> At least, so to speak.

<sup>29</sup> This is an “empirical” market portfolio (like a stock market index). Not necessarily the theoretical market portfolio based on the CAPM-Assumptions.

<sup>30</sup> No-arbitrage conditions.

<sup>31</sup> At least, so to speak.

<sup>32</sup> See for an explanation of the terms and their relationship,

Friedrich, 2015, pp. 13.

<sup>33</sup> It is therefore a “real” investment opportunity and not a model construct, such as the market portfolio at Markowitz (1952) or within the framework of the CAPM. The assumption that the (empirical) market portfolio can be invested corresponds to the idea of “availability” in Richter, 2005, p. 22.

CAPM), downside risk measures can also be used. With these downside risk measures, risk is expressed as “possible loss” or the utilization of a risk coverage potential (equity and liquidity reserve) that is scarce in reality. The risk measure should be homogeneous and translation- or position-invariant<sup>34</sup>.

$$(5) \quad R(\widetilde{CF}_A) = R(x \cdot (1 + \tilde{r}_M) + y \cdot (1 + r_f))$$

The expected value of the repayment of the investment in the market portfolio and the risk-free investment should correspond to the expected value  $E(\widetilde{CF}_A)$ .

$$(6) \quad E(\widetilde{CF}_A) = E(x \cdot (1 + \tilde{r}_M) + y \cdot (1 + r_f)) = x \cdot (1 + E(\tilde{r}_M)) + y \cdot (1 + r_f)$$

The value of the risky cash flow  $\widetilde{CF}_A$  corresponds to the sum of the two investments  $x$  and  $y$ . The same risk and the same expected value imply the same value.

$$(7) \quad Value(\widetilde{CF}_A) = x + y$$

The replication equation can be derived from equations (6) and (7)<sup>35</sup>.

$$(8) \quad R(\widetilde{CF}_A) = R(x \cdot (\tilde{r}_M - E(\tilde{r}_M)) + E(\widetilde{CF}_A))$$

If the risk measure is known, this equation can be solved and thus evaluated. It is important to know whether this is a position-dependent risk measure (such as the value-at-risk or the conditional value-at-risk) or a position-independent risk measure (such as the standard deviation or the deviation value-at-risk). The deviation value-at-risk or relative value-at-risk is defined as  $DVaR_\alpha(\widetilde{CF}_A) = E(\widetilde{CF}_A) + VaR_\alpha(\widetilde{CF}_A)$  with  $\alpha$  as confidence level (e.g.  $\alpha=99\%$ ).

Since cash flows often cannot be described by normal or log-normal distributions (e.g., because of fat tails), downside risk measures are gaining in importance.

In the following, only position-independent risk measures such as standard deviation are considered more closely, since they are seen as a measure of planning reliability or the extent of possible plan deviations (from the expected value)<sup>36</sup>. This applies to these (see *Rockafellar/Uryasev/Zabarankin, 2002*):

$$(9) \quad R(a + b \cdot \widetilde{CF}_A) = b \cdot R(\widetilde{CF}_A)$$

With equation (9) equation (8) simplifies to

$$(10) \quad R(\widetilde{CF}_A) = x \cdot R(\tilde{r}_M)$$

For the value one obtains by transformations (and by neglecting a time index)<sup>37</sup>

$$(11) \quad Value(\widetilde{CF}_A) = \frac{E(\widetilde{CF}_A) - \frac{E(\tilde{r}_M) - r_f}{R(\tilde{r}_M)} R(\widetilde{CF}_A)}{1 + r_f} = \frac{E(\widetilde{CF}_A) - \lambda_{CE} \cdot R(\widetilde{CF}_A)}{1 + r_f}$$

$$\text{with } \lambda_{CE} = \frac{E(\tilde{r}_M) - r_f}{R(\tilde{r}_M)}$$

A special variant of equation (3) has thus been derived<sup>38</sup>. The market price of the risk  $\lambda$  shows how much more return per unit of risk can be expected for the alternative investments under consideration.

In the simplest case, the risk discount corresponds to the product of the risk premium and the risk volume (e.g. “equity requirement” as a risk measure based on value-at-risk).

Now assume the risk measure  $R(\widetilde{CF}_A)$  is the standard deviation and so  $R(a + b \cdot \widetilde{CF}_A) = b \cdot \sigma(\widetilde{CF}_A)$ . Now the following equation shows how is the value of the cash flow  $\widetilde{CF}_A$ .

$$(12) \quad Value_0(\widetilde{CF}_A) = x + y = \frac{E(\widetilde{CF}_A) - \sigma(\widetilde{CF}_A) \cdot \frac{E(\tilde{r}_M) - r_f}{\sigma(\tilde{r}_M)}}{1 + r_f}$$

The cost of capital ( $c$ ) is thus implicitly the ratio of the cost of  $E(\widetilde{CF}_A)$  to  $Value_0(\widetilde{CF}_A)$  which will be discussed later in section 4.2.

Until now, it has been assumed that the cash flow from investment  $A$  and the market portfolio is fully correlated, i.e., that the correlation coefficient  $\rho_{AM} = 1$  or investment  $A$  is the only asset.

As a rule, however, this assumption will not be fulfilled and thus diversification possibilities will be available so that only the non-diversifiable portion of the risk (the systematic risk) of the cash flow is relevant for the valuation.

This reduces the valuation-relevant risk of the cash flow by multiplying the standard deviation by  $\rho_{AM} = \rho(\widetilde{CF}_A, \tilde{r}_M)$ , so that the following equation results:

$$(13) \quad Value_0(\widetilde{CF}_A) = \frac{E(\widetilde{CF}_A) - \rho_{AM} \cdot \sigma(\widetilde{CF}_A) \cdot \frac{E(\tilde{r}_M) - r_f}{\sigma(\tilde{r}_M)}}{1 + r_f} = \frac{E(\widetilde{CF}_A) - \lambda \cdot \sigma(\widetilde{CF}_A) \cdot \rho_{AM}}{1 + r_f}$$

Equation (14) corresponds to the certain equivalence equation of the CAPM (3)<sup>39</sup>. The following conditions apply:

<sup>34</sup> Position-independent. See *Dorfleitner/Gleißner, 2018*.

<sup>35</sup> See *Dorfleitner/Gleißner, 2018*.

<sup>36</sup> See *Dorfleitner/Gleißner, 2018* for translation- invariant risk measures.

<sup>37</sup>  $\widetilde{CF} = \widetilde{CF}_1$  is considered to be the cash flow of period 1. Period 1 is between time  $t=0$  and  $t=1$ . Valuation date is  $t=0$ .

<sup>38</sup> See *Gleißner/Wolfrum, 2009*.

<sup>39</sup> See *Robichek/Myers, 1966; Rubinstein, 1973*.

- the risk measure is the standard deviation,
- only non-diversifiable, systematic risks are assessed, and
- there are homogeneous expectations, i.e., the cash flow is valued by the capital market according to the planning ( $\sigma(\widetilde{CF}_A) = \sigma_A$ ).

It should be noted that the replication equations do not conflict with CAPM if the same assumptions are made as in CAPM and in this case the risk measure (“capital requirement”, CVaR or VaR) contains exactly the same information as the standard deviation and the beta factor (see *Mai*, 2006, on the relationship with the traditional CAPM return equation, specifically on the assumption of proportionality of cash flow and value fluctuations).

The replication methodology can also be extended to multi-period cash flows<sup>40</sup>.

#### 4.2 Deriving the cost of capital from the valuation equation using incomplete replication

The procedures described in section 4.1 allow the risk-adjusted measurement of uncertain cash flows (in one or more periods). However, valuation using a risk discount in the numerator, i.e., the calculation of certainty equivalents, is unusual in valuation practice. The previously explained (semi-investment theoretical) valuation based on “incomplete replication” can, however, also be directly linked to the discounted cash flow (DCF) methods known in practice. For this purpose, it is necessary to determine the cost of capital (discount rate) of the DCF methods using the methods explained in section 4.1.

The bridge from the aggregated total risk, e.g. expressed by the standard deviation of the cash flow  $\sigma(\widetilde{CF})$ , to the company value, is precisely the cost of capital (or certainty equivalents). In contrast to the traditional “capital market-oriented” valuation, the cost of capital in a risk simulation can be derived directly from the earnings risk and not from historical stock return fluctuations (as is usually the case with the beta factor of the CAPM; see *Gleißner*, 2011 and 2014). The results of a risk analysis are used, on the one hand, to obtain expected cash flow values and, on the other hand, to derive the cost of capital rates consistently (the consistency between the expected value of the cash flows in the numerator and the cost of capital rates in the denominator is a notable advantage of the methodology explained). Such a discount rate, which is often assumed to be constant, can be derived

as a risk measure from the standard deviation of the cash flow, for example. It obviously applies:

$$(14) \quad Value_0(\widetilde{CF}_A) = \frac{E(\widetilde{CF}_A)}{1+c}$$

If one resolves this equation with equation (14) for the value  $V$  after  $c$ , one obtains the risk-adequate cost of capital. If  $\widetilde{CF}$  is the operating free cash flow (oFCF),  $c$  is the weighted cost of capital (WACC)); if  $\widetilde{CF}$  is the flow to equity (FTE),  $c$  is the cost of equity.

Based on the risk-free interest rate, the following equation for the risk-adequate capitalization rate (cost of capital) is obtained<sup>41</sup>:

$$(15) \quad c = \frac{1+r_f}{1-\lambda \cdot \frac{\sigma(\widetilde{CF}_A)}{E(\widetilde{CF}_A)} \cdot d} - 1 = \frac{1+r_f}{1-\lambda \cdot V \cdot d} - 1$$

The ratio of cash flow risk  $\sigma(\widetilde{CF}_A)$  to expected cash flow  $E(\widetilde{CF}_A)$  is the coefficient of variation  $V$ . The variable  $\lambda$  shows the excess return per unit of risk (*Sharpe Ratio*).

$$(16) \quad \lambda = \frac{\text{Market Risk Premium}}{\sigma(\tilde{r}_m)} = \frac{E(\tilde{r}_m) - r_f}{\sigma(\tilde{r}_m)}$$

$\lambda$  is dependent on the expected return of the market index, its standard deviation and the risk-free rate of return and expresses the risk/return profile of the alternative investments: to value means to compare (*Moxter*, 1983). As the owners do not necessarily bear all the risks of the company, the risk diversification factor  $d$  must also be taken into account. It shows the proportion of risks of a company that the owner has to bear in equation (16)<sup>42</sup>.

An estimate of the degree of risk diversification  $d$  can be derived by the correlation of the (trend-adjusted) earnings (or earnings growth) of the company to the earnings of all companies in the market index. The risk diversification factor  $d$  implicitly follows from the simulation-based risk aggregation if exogenous risk factors are considered independently to record the systematic, cross-company risk<sup>43</sup>. Under the special assumptions of the CAPM,  $d$  conforms to a correlation with the return on the market portfolio.

Equation (16) can be used for different definitions of cash flows  $\widetilde{CF}_A$ . If flow to equity is used as cash flow, the cost of equity is obtained. If the operating free cash flow is used, the weighted average cost of capital (WACC) is obtained. The WACC results

<sup>40</sup> See *Dorfleitner/Gleißner*, 2018.

<sup>41</sup> For  $\lambda \cdot \frac{\sigma(\widetilde{CF}_A)}{E(\widetilde{CF}_A)} \cdot d < 1$

<sup>42</sup> It is the proportion of  $R(\widetilde{CF})$  to additional (“incremental”) risk

in the portfolio of the owner caused by the company (see *Gleißner*, 2011 and *Tasche/Tibiletti*, 2003).

<sup>43</sup> “Risk factor model”; see *Gleißner*, 2017a, pp. 261-263.



“directly”, without first having to calculate cost of equity and cost of debt and weight them appropriately. The total extent of the risks determines the total cost of capital (only in a second step are the total risks divided between equity and debt capital providers, which determine the cost of equity and cost of debt). Determining the total cost of capital (WACC) in this way is comparatively simple and there is no need for leveraging or deleveraging when calculating the cost of equity.

It should be noted that it is not necessary to calculate the cost of capital only for a representative period and to assume it to be constant for all periods for the sake of simplicity. Of course, it is also possible to calculate periodic cost of capital. In addition to periodic cost of capital, it is also possible, and useful in many valuation cases, to calculate two different cost of capital: a cost of capital  $c_1$  for the detailed planning period and a cost of capital  $c_2$  for the continuation period. This is particularly appropriate if, in the detailed planning period, the risk-return profile of the company, and thus the coefficient of variation  $V$ , still differ significantly from that in the continuation period. This is particularly the case if, for example, a young company has significantly higher risks at the beginning of its existence than later when it is established (i.e. in the continuation period).

#### 4.3 Risk analysis and risk aggregation using Monte Carlo simulation

The identification and quantification of the company's risks (opportunities and threats) must be the basis for the risk-appropriate evaluation of a company.

As a result, the risk analysis and risk aggregation - as shown above - leads to cost of capital that express the risk-adjusted requirement for the return on a project, business unit or company (e.g., for the calculation of a discounted cash flow DCF or Economic Value Added EVA). In addition to the risk measure of the standard deviation, which is based on a normal distribution and is used in CAPM, there are other risk measures. These risk measures are often better suited to describe the actual risk in the company. In order to determine suitable risk measures for company valuation, the actual risks in the company must be determined. This is done with the help of a risk analysis. Then it has to be examined how the risks are related to each other and how they affect the cash flows and thus the company value. This is done on the basis of a Monte Carlo simulation. The results of the Monte Carlo simulation can be used to calculate suitable risk measures. These risk measures are then incorporated into the company valuation using the certainty equivalence method and are expressed by the variable  $R(CF)$

##### 4.3.1 Risk analysis of corporate risks

The first step in risk analysis is the identification of risks, which can be structured as follows:

###### (1) Strategy and strategic risks

Strategic risks are the risks arising from the threat to the company's most significant potential for success.

###### (2) Controlling, operational planning and budgeting risks

In controlling, business planning or budgeting, certain assumptions are made (for example, with regard to the growth rate of the economy, exchange rates and successes in sales activities). All uncertain planning assumptions show a risk because plan deviations can occur. The causes of plan deviations show the effects of existing risks.

###### (3) Risk workshops (risk assessment) on performance risks

Certain types of risk are best identified in a workshop through critical discussions. These include, in particular, operational risks, legal risks, political risks, and risks arising from support services (e.g., IT).

For the quantitative description of a risk, a probability distribution can be used that describes the effects of a risk on earnings in a period (e.g., year). A more differentiated consideration is possible if a risk is described by (1) a probability distribution for the frequency of the occurrence of the risk in a period and (2) a probability distribution for the amount of damage per occurred risk event.

##### 4.3.2 Risk aggregation using Monte Carlo simulation

It is not individual risks but the aggregated overall risk scope that is decisive for assessing a company's (free) risk-bearing capacity and the degree of threat to its continued existence. Aggregation across all individual risks and over time is therefore necessary. Since only quantified risks can be aggregated, all relevant risks must be quantified. By aggregating the quantified risks in the context of planning, it is examined what effects these have on future earnings, future cash flows, the key financial indicators, credit agreements (covenants), the rating, and thus on the enterprise value. For example, it is necessary to calculate the probability that risks (e.g., an economic downturn in connection with a failed investment project) could cause the company's future rating to fall below a level (B rating) necessary for the company's ability to service its debt.

The aggregation of risks in the context of corporate planning requires the use of simulation methods (Monte Carlo simulation) because risks - unlike costs - cannot be added together, at least if special cases (normal distributions) are excluded. Furthermore, risks in an integrated planning model must also be aggregated over several years to identify serious crises over

time. Simulation methods are the further development of the well-known scenario analysis techniques<sup>44</sup>. Monte Carlo simulation is used to analyze a large representative number of risk-related possible future scenarios (planning scenarios) in risk aggregation. In this way, a frequency distribution and thus a realistic range of future cash flows and returns are shown, i.e., the planning reliability or extent of possible negative deviations from the plan.

### 4.3.3 Risk measures

In addition to the quantitative description of risks, the calculation of risk measures (R) is another sub-task in risk quantification<sup>45</sup>. The term risk measure is a collective term for statistical measures that make it possible to describe the uncertainty of an event quantitatively. A risk measure maps a frequency or probability distribution to a real number. A risk measure expresses the scope of risk of a distribution in a number that can then be used for further economic and application-oriented calculations. Risk measures are necessary to enable simple “calculating with risks” (as shown in section 4). They thus serve to transform risk or uncertainty.

A distinction is made between position-dependent (position-invariant) and position-independent risk measures. Position-dependent risk measures, such as the value at risk, are dependent on the expected value. If a position-dependent risk measure is not applied to a random variable  $\tilde{X}$ , but to a centered random variable  $\tilde{X} - E(\tilde{X})$ , the result is a position-independent risk measure<sup>46</sup>. Position-independent risk measures (such as the standard deviation or deviation value at risk (DVaR)) describe the extent of plan deviations and are therefore also referred to as deviation measures.

Furthermore, a distinction is made between one-sided and two-sided risk measures. Two-sided risk measures measure deviations from the planned or expected value in both directions, i.e., opportunities and risks. The one-sided risk measures consider only possible deviations in one direction, mostly possible negative plan deviations.

For the derivation of the evaluation equations, it is assumed, as explained above, that the risk measure (a) is homogeneous and (b) is either translational or position-invariant, and therefore the following applies accordingly:

- positive homogeneity (PH) is defined by  $R(a\tilde{X}) = aR(\tilde{X})$ ,
- translation invariance (TI) is defined by  $R(\tilde{X} + a) = R(\tilde{X}) - a$ ,

- position invariance (PI) is defined by  $R(\tilde{X} + a) = R(\tilde{X})$

with  $a \in \mathbb{R}$  and  $a > 0$

## 5. Insolvency risk and rating

Previously, this section explained how the risks (opportunities and threats) affect the expected value of cash flows and the cost of capital. In real, incomplete capital markets with rating and financing restrictions, there is a further impact of risks that is discussed below. A particularly unfavorable combination of individual risks can arise scenarios that lead to the insolvency of the company and thus to the interruption of the cash flow of the (previous) owners. This risk of insolvency has so far received little attention in valuation practice, although it can have considerable effects on the value of the company.

It should be noted that the insolvency risk, especially the probability of insolvency  $p$ , influences the expected value of the cash flows and their development over time<sup>47</sup>.

In the detailed planning phase, the probability of insolvency must be taken into account directly when determining the expected values (as a scenario with, as a rule, no return to the owners). In general, it is advisable to map insolvency scenarios in detail in a stochastic event space or in the paths of a simulation model even in the continuation phase.

In addition to considering the insolvency scenario in the detailed planning, it should be noted that insolvency can occur in any year of the continuation phase. An approach that is partly implemented in valuation practice is the evaluation of an insolvency scenario for the expected result. Even if this may already sensitize to the possibility of insolvency, considerable problems remain: On the one hand, the estimated probability of insolvency is usually not rating and planning consistent, on the other hand, it is often ignored that insolvency is possible every year, so that there are many insolvency scenarios - and in the long term, insolvency is a scenario with a high probability.

If it is assumed for the continuation phase when determining the terminal value that the probability of insolvency - corresponding to the steady state in the terminal value formula - remains constant, it leads (under otherwise identical conditions) over time to continuously declining expected cash flows.

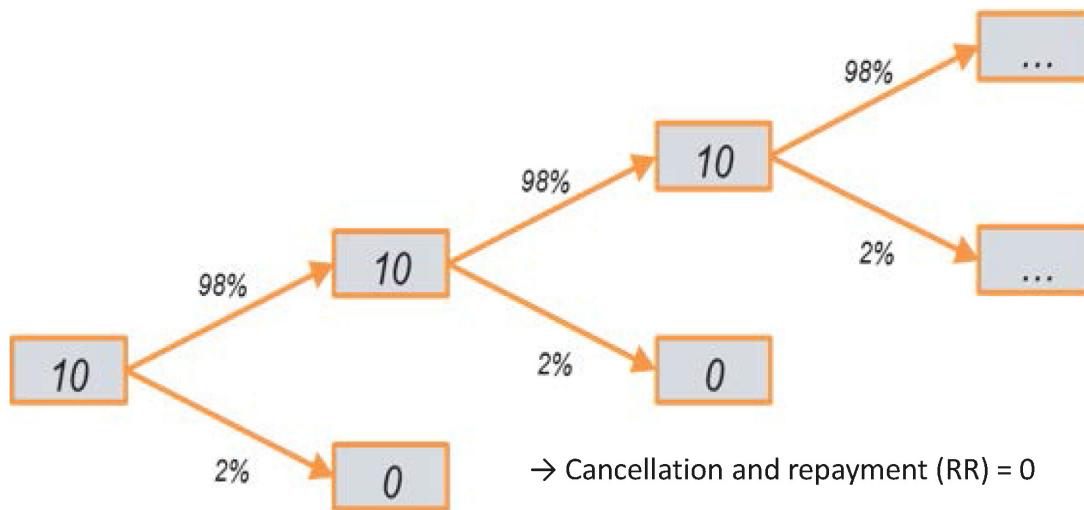
<sup>44</sup> See *Grisar/Meyer*, 2015 and 2016 on significance.

<sup>45</sup> See cf. *Gleißner*, 2017a and *Artzner et al.*, 1999, *Pedersen/Satchell*, 1998, *Albrecht/Maurer*, 2005 and *Brandtner*, 2012.

<sup>46</sup> See *Pedersen/Satchell*, 1998.

<sup>47</sup> See *Gleißner*, 2010 and *Friedrich*, 2015 and 2016 and *Lahmann/Schreiter/Schwetzler*, 2018.

Graph 2: Implications of a probability of defaults (p=2%)



In the long term - in the continuation phase -  $p$  has the effect of a negative growth rate<sup>48</sup> (see graph 2), which must be taken into account when calculating the terminal value (TV)<sup>49</sup>. This applies here:

$$(17) \quad Value^{TV} = \frac{E(\widetilde{CF}) \cdot (1-p)}{c+p}$$

This also applies if cost of capital or discount rates ( $c$ ) are calculated according to the CAPM.

With a growth rate<sup>50</sup> ( $g$ ), the (conditional) expected values of the cash flows  $E(\widetilde{CF})$ <sup>51</sup> and a discount rate ( $c$ ), the following equation results for the company value (Value) in the continuation phase (terminal value) as a function of the insolvency probability ( $p$ )<sup>52</sup>:

$$(18) \quad Value = \sum_{t=0}^{\infty} \frac{E(\widetilde{CF}) \cdot (1-p) \cdot (1+g)^t}{(1+c)^t} = \frac{E(\widetilde{CF}) \cdot (1-p) \cdot (1+g)}{c-g+p \cdot (1+g)}$$

The value of a company (or its terminal value) with  $g = 0$  is then:

$$(19) \quad Value = \frac{E(\widetilde{CF}) \cdot (1-p)}{c+p}$$

When determining an infinite series (Gordon Shapiro model), the insolvency probability (just like the growth rate) actually appears in the numerator in each individual period (see equation (17)). However, the dissolu-

tion of the series leads to the fact that the probability of insolvency (as well as the growth rate) mathematically “migrates” into the denominator. This does not mean, however, that double counting would occur or that the probability of insolvency would become a component of the discount rate. In the continuation phase, the probability of insolvency thus largely acts like a “negative growth rate” - but is not part of the cost of capital.

Anyone who accepts the recording of a growth rate in the terminal value must also accept the consideration of the probability of insolvency derived from the same assumption system. The above-mentioned “pragmatic” recording of the possibility of insolvency within the framework of the usual (deterministic) “terminal value formula” is not without alternatives. A more precise recording of the risks and stochastic dependencies, also between the individual periods, can be achieved e.g. by binomial models (Friedrich, 2015)<sup>53</sup> and especially by flexible stochastic planning models and Monte Carlo simulation. When calculating the expected values in the simulation, the insolvency scenarios are recorded and a closed “terminal value formula” is practically unnecessary if one simulates many years of the future. Nevertheless, as explained above, pragmatic solutions certainly also have practical advantages.

<sup>48</sup> See Shaffer, 2006, Gleißner, 2010; Knabe, 2012; Saha/Malkiel, 2012; Ihlau/Duscha, 2019.

<sup>49</sup> See Gleißner, 2017c, Knabe, 2012 and Saha/Malkiel, 2012 and Lahmann/Schreiter/Schwetler, 2018.

<sup>50</sup> On the relationship between  $w$  and  $c$  in inflation-, accumulation- and tax-indexed (endogenous) growth see Tschöpel/Wiese/Willershausen, 2010.

<sup>51</sup> Without insolvency (conditional expected value) and period-invariant probability of insolvency (here for  $T$ , i.e. after detailed planning phase).

<sup>52</sup>  $E(\widetilde{CF})$  is the expected value of growth and probability of insol-

vency. If  $E(\widetilde{CF})$  is interpreted as cash flow before probability of insolvency,  $(1 + g)$  is omitted.

<sup>53</sup> In addition, one can immediately see with binomial models by Friedrich, 2015, that, as is usual with such (simple) binomial models, no negative free cash flows can occur, which is unrealistic. Insolvencies naturally occur especially with negative free cash flows. The impossibility of depicting negative cash flows in the simple binomial model results from the fact that in the binomial tree the last cash flow is multiplied by 1.4 (up scenario) with a previously given probability (e.g.  $p = 60\%$ ) or by 0.8 (down scenario) with a probability of  $(1 - p)$ .

**6. Case study: From CAPM to risk-adequate assessment**

6.1 Introduction

The explanations above will be illustrated in the following with a small example. The transition from “traditional” planning, which here is based on the assumption of (ambitious) planned values of the company and discount rates calculated using CAPM, takes place in three steps.

1. The systematic analysis of existing risks allows a transparent reconciliation of the usual planned values with the expected values relevant to valuation, which will be realised “on average”. This creates transparency with regard to the essential, even uncertain planning assumptions and an adequate consideration of a risk overhang.

2. The probability of insolvency expressed by the rating can be assessed by means of a key financial figures rating and the evaluation of combined effects of risks (Monte Carlo simulation). The often ignored value driver “probability of insolvency (insolvency risk)” is taken into account in the implications for the company value and thus takes into account the fact that, contrary to the usual assumption, companies do not exist forever (see section 5).

3. The transparency created by risk analysis and risk simulation (risk aggregation) with regard to planning security and thus the aggregated cash flow risk (cash flow volatility) makes it possible to derive risk-adjusted cost of capital. Expected values of cash flows (“numerator”) and discount interest rate (“denominator”) are thus determined consistently and the problems of the low informative capacity of CAPM cost of capital (due to capital market imperfections) are avoided. This enables a risk-adjusted valuation, i.e. a calculation taking into account the risks of a company’s future earnings and cash flows.

6.2 Initial Situation: CAPM and planning values (corporate planning)

The valuation of the company is based on a two-year detailed planning period (t = 1,2) whereby the second period is regarded as representative for the future<sup>54</sup>. The enterprise has planned the cash flow to equity to be discounted. The long-term growth rate is assumed to be g = 0 and insolvency risks are neglected. The following assumptions are made about the parameters of the environment:

- r<sub>f</sub> = 3% (for all periods)

- r<sub>m</sub><sup>e</sup> = 5% (market risk premium)
- β = 0.75 (calculated with a market price of the risk λ = 0.25)<sup>55</sup>

From the information provided, the following time-invariant cost of capital results.

$$(23) \quad c = r_f + \beta \cdot (r_m^e - r_f) = 3\% + 0.75 \cdot 5\% = 6.75\%$$

The following applies to the value

$$(24) \quad \text{Value} = \sum_{t=1}^{\infty} \frac{E(\overline{CF}_t)}{(1+c)^t}$$

**Table 1: Company valuation based on planned values and CAPM**

|                     | 1     | 2     | TV                   | NPV of the cash flows and TV |
|---------------------|-------|-------|----------------------|------------------------------|
| T                   |       |       |                      |                              |
| Cash flow (planned) | 10    | 15    | (15 ...)             |                              |
| c (CAPM, Beta)      | 6.75% | 6.75% | 6.75%                |                              |
| Value               | 9.37  | 13.16 | 195.01 <sup>56</sup> | <b>217.54</b>                |

On the basis of the cash flows and terminal value shown in table 1, the company value is calculated as

$$(25) \quad \text{Value}_1 = 217.54 \text{ Euro}$$

6.3 First step: Transfer from plan values to expected values

The discounted cash flow methods are based on expected cash flows. In order to calculate these, the results of the analysis of chances and risks of the company are used. In particular, uncertain planning assumptions, which form the basis for the cash flow forecast in table 1, are considered and described using appropriate probability distributions. Without further explanation of details, it is assumed that risk analysis and risk aggregation (Monte Carlo simulation) result in a threats overhang and thus lower expected values compared with the planned values.

All other information is unchanged, i.e. the cost of capital rate c = 6.75% derived from CAPM is still used. The Monte Carlo simulation also produces a quantification of the cash flow risk, in the example here a coefficient of variation of V = 0.35, which, however, is not (yet) included in the valuation (see step 3 in 6.5).

<sup>54</sup> For t = 3, 4, ..., 8.

<sup>55</sup> It shall apply to company i: c = r<sub>f</sub> + λ · σ<sub>i</sub> · ρ with σ<sub>i</sub> as the standard deviation of the stock return of i.

<sup>56</sup> The present value in t = 0 of the TV in t = 2 is 195.01 = 15 / ((0.0675)(1 + 0,0675)<sup>2</sup>).

**Table 2: Company valuation based on expected values and CAPM**

| T                    | 1     | 2     | TV     | NPV of the cash flows and TV |
|----------------------|-------|-------|--------|------------------------------|
| Cash flow (planned)  | 10    | 15    | 15     |                              |
| Cash flow (expected) | 9     | 13    | 13     |                              |
| c (CAPM, Beta)       | 6.75% | 6.75% | 6.75%  |                              |
| Value                | 8.43  | 11.41 | 169.01 | <b>188.85</b>                |

Taking into account the effects of the opportunities and threats on the expected value of the cash flows, the resulting value is now  $Value_2 = 188.85$

6.4 Second step: Consideration of the effects of insolvency risk

In step 1, the company’s earnings risks were taken into account. However, no account was taken of the fact that risk-related future scenarios could arise for the company, which could lead to insolvency and thus to the discontinuation of the cash flows for the owners (as the valuation subject). Now it is taken into account that insolvency risks influence both the expected value of the cash flows in each period of the detailed planning phase and the expected value in the continuation phase ( $t > 2$ ). Further effects of the insolvency risk, e.g. on the tax shield, are neglected. It is also assumed that the implications of the probability of insolvency  $p$  expressed by the rating are already included in the interest rates and thus in the cost of debt (and thus in the expected values of the cash flows). In general, it is also necessary to adjust interest rates and cost of debt to the rating.

In the case study, the probability of insolvency  $p$  is estimated based on financial ratios, i.e. equity ratio 25% and return on capital employed 10%. (The Monte Carlo simulation carried out for risk aggregation serves to check the plausibility of the probability of insolvency). Furthermore, an insolvency probability of  $p = 1.55\%$  is assumed<sup>57</sup>.

This results in the following company valuation:

**Table 3: Company valuation based on expected values, CAPM, and insolvency risk**

| T                                           | 1      | 2      | TV     | NPV of the cash flows and TV |
|---------------------------------------------|--------|--------|--------|------------------------------|
| Cash flow (planned)                         | 10     | 15     | 15     |                              |
| Cash flow (expected)                        | 9      | 13     | 13     |                              |
| probability of survival                     | 98.45% | 96.92% | 95.42% | sinking with p               |
| Cash flow (expected, incl. insolvency risk) | 8.86   | 12.60  | 12.40  |                              |
| c (CAPM, Beta)                              | 6.75%  | 6.75%  | 6.75%  |                              |
| Value                                       | 8.43   | 11.41  | 131.15 | <b>150.51</b>                |

The company value is reduced to  $Value_3 = 150.51$  Euro due to the consideration of insolvency risk.

6.5 Third step: Calculation of cost of capital based on earnings risk (coefficient of variation of earnings)

As already mentioned, the coefficient of variation of the returns is - according to the simulation -  $V = 35\%$ . The risk diversification factor here is  $d = 0.5$ , which corresponds precisely to the correlation between the return on the shares of the valuation object and the return on the market portfolio.

With the results from risk analysis and risk simulation in step 1, the coefficient of variation of the returns was calculated in addition to the adjustment of the planned values, but has not yet been taken into account. The coefficient of variation is a measure of the overall scope of risk (extent of possible deviations from the plan) With equation (15) explained above, information about the risks of the company - instead of information about the risks of the company’s shares - is now used as the basis for deriving the discount rate. The following applies accordingly

$$(26) \quad c = \frac{1+r_f}{1-\lambda \cdot V \cdot d} - 1 = \frac{1+0.03}{1-0.25 \cdot 0.35 \cdot 0.5} - 1 = 7.71\%$$

In this third step, the company value is now determined with a cost of capital  $c$  corresponding to the earnings risks.

<sup>57</sup> It is based on an empirically determined simple formula for estimating the probability of insolvency

$$P = \frac{0.265}{1 + e^{-0.41+7.42 \cdot \text{equity ratio}+11.2 \cdot ROCE}} \text{ (see Gleißner, 2017a, pp. 336-338 with reference to the basic research projects).}$$

**Table 4: The final valuation**

|                                             | 1           | 2            | TV            | NPV of the cash flows and TV |
|---------------------------------------------|-------------|--------------|---------------|------------------------------|
| T                                           |             |              |               |                              |
| Cash flow (planned)                         | 10          | 15           | 15            |                              |
| Cash flow (expected)                        | 9           | 13           | 13            |                              |
| probability of survival                     | 98.45%      | 96.92%       | 95.42%        | sinking with p               |
| Cash flow (expected, incl. insolvency risk) | 8.86        | 12.60        | 12.40         |                              |
| c (earnings risks V)                        | 7.71%       | 7.71%        | 7.71%         |                              |
| <b>Value</b>                                | <b>8.23</b> | <b>10.86</b> | <b>115.47</b> | <b>134.56</b>                |

The resulting value is now  $Value_4 = 134.56$  Euros.

Now the information on the risk profile of the company as a whole is adequately taken into account in the company valuation. It should be mentioned that the adjustment according to steps 1 and 2 is also necessary if the valuation of perfect capital markets and in particular the validity of the assumptions of the CAPM are assumed.

In the example, in comparison to the initial situation, the enterprise value decreases with every further step. This is not necessarily the case. Thus, there are constellations in which existing opportunities outweigh existing dangers and thus the expected value is higher than a (conservative) plan value. The consideration of the probability of insolvency ( $p$ ), contrary to the first impression, does not necessarily lead to a lower enterprise value. This is because, in valuation practice, the growth rate  $g$  applied in view of economic growth for a company's long-term profit growth (in the continuation phase) is implicitly offset by a 'typed' probability of default ( $\bar{p}'$ ).

Empirical studies<sup>58</sup> show typical growth rates in the order of 0 to 0.5% in the continuation phase. This is much less than the inflation rate alone (excluding real economic growth) and can only be explained by assuming it as an "insolvency-risk-adjusted" growth rate with a typical probability of insolvency (of, for example, 1%) already deducted. The implication for the valuation of different companies is clear: if implicit (and non-transparent) is valued with a medium probability of default, which is offset against the growth rate, it leads to advantages and disadvantages for certain companies: companies with a below-average prob-

ability of default have a higher value compared with the traditional approach. The approach tends to be too low, while those with an increased probability of default are too high. *Campbell, Hilscher and Szilagyi (2008)* show, for example, that companies with a very good rating on the stock exchange generate above-average returns that can be explained if one assumes that the probability of default is ignored, especially in the valuation calculus of most capital market participants, and thus "quality companies" with a very good rating tend to be undervalued and accordingly generate above-average risk-adjusted returns).

### 7. Summary and outlook

In practice, there are many problems with the valuation of companies, for example due to the often unjustified assumption of perfect capital markets. With risk analysis, Monte Carlo simulation and the method of incomplete replication, instruments exist that take account of the imperfections of the capital market and can also be applied to companies that are not listed on the stock exchange. The valuation-relevant risks are derived by means of risk analysis and risk aggregation, and planning consistency - e.g. via standard deviation or VaR as risk measure - is recorded in the valuation, whereby financing restrictions of the creditors can be taken into account. The detour of obtaining risk information from historical stock returns - instead of from the company itself - is avoided.

Even if CAPM-based valuation is to be applied, the "risk discount variant of CAPM" and the information provided by the risk analysis can be used to ensure that the appraiser is not dependent on historical stock returns that are often missing or not representative for the future. In this respect, the valuation approach also contributes to a new (more accurate) interpretation of the paradigm of value orientation (value based management): orientation towards the interests of the owners, but use of the best available information - and these are not always those of the capital market.

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