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# An international agreement with full participation to tackle the stock of greenhouse gases

by

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# An international agreement with full participation to tackle the stock of greenhouse gases

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**Abstract**

This paper analyzes greenhouse gas emissions that build up an atmospheric stock which depreciates over time. Weakly renegotiation-proof and subgame perfect equilibria in a game of international emission reduction exist if countries put a sufficiently high weight on future payoffs, even though there is a discontinuity in the required discount factor due to the integrity of the number of punishing countries. Treaties are easier to reach if the gas depreciates slowly.

Keywords: global warming; international agreement; weak renegotiation-proofness

JEL: Q54; F53; H41

## 1 Introduction

International negotiations aiming at mitigating global warming often fail. No country has to participate in a climate agreement and all countries can renegotiate at any time, especially if governments change due to regular elections. Therefore, a self-enforcing climate agreement has to be both individually and collectively rational.

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Collective action concerning climate policy has been analyzed by several game-theoretical contributions applying dynamic models of international environmental public good provision. Barrett (1999, 2002), Asheim et al. (2006), Froyn and Hovi (2008), Asheim and Holtsmark (2009) as well as Kratzsch et al. (2010) consider an infinitely repeated $N$-player prisoner's dilemma game, with a stage game giving each country the choice between participating in an agreement that induces an emission reduction and rejecting the agreement. All authors assume that payoffs are identical in each stage game, i.e., there is no lasting effect of a one-period emission over time. In the context of global warming, these approaches appear appropriate if the emitted greenhouse gas possesses only a short atmospheric lifetime, as for example methane.

However, most greenhouse gases that contribute to global warming last for a long period of time (Houghten et al. 1995, p. 22): Emissions in period $t$ build up an atmospheric stock that depreciates slowly over time. Therefore, to treat the problem of greenhouse gas emissions properly, payoffs in each future period have to depend on all former emissions by all countries.

Full participation in a climate agreement is essential to mitigate global warming effectively. If only a subset of countries participates in an agreement while the world supply of fossil fuels remains fixed, reductions in the demand for fossil fuels will merely lower the world price of carbon and provoke non-participating countries to consume what the participating countries have saved (Sinn 2008). The so-called "green paradox" sows seeds of doubt about the benefit of partial agreements.

This paper explicitly formalizes the stock of one emitted greenhouse gas with a long lifetime like carbon dioxide. By approximating the cost of global warming depending on the concentration of atmospheric carbon dioxide, we can identify equilibria in the greenhouse gas stock game where all countries are willing to sign a climate agreement and reduce greenhouse gas emissions.

## 2 The model

We consider a world consisting of $i = 1, ..., N$ identical countries. The countries face a public bad that is represented by the total stock $G$ of greenhouse gases (GHGs) beyond its natural level, normalized to 0. Let $0 < q < 1$ be the share of the existing stock of GHGs that is not dissipated from the atmosphere within a period and hence still remains in the atmosphere in the following period. Each country emits $g_{i,t}$ units of GHGs during period $t$. Following Dutta and Radner (2009), the law of motion for the global stock of GHGs is

$$G_t = qG_{t-1} + \sum_{i=1}^{N} g_{i,t}. \tag{1}$$

The costs of global warming to a country in period $t$ depend on the total stock of GHGs and are approximated by

$$w_t(G_t) = G_t. \tag{2}$$

The costs are increasing with a higher stock of GHGs due to a costly change in the associated climatic conditions. Like Dutta and Radner (2009), we assume linear costs of climate change, although, in reality, these costs can become very non-linear, for example in the case of non-linear feedback effects such as the melting of the polar ice caps or the leakage of methane induced by permafrost melting. However, there is little scientific consensus on the correct form of non-linearity in climate costs, and to the extent that the world might continue to stay in the linear cost segment for the next 50 or 100 years – which appears to be the more relevant time-frame due to discounting – our results can be seen as approximations of a non-linear model.

In every period of the infinitely repeated game, each country has to determine its level of GHG emissions, which is given by

$$g_{i,t} = (1 - \alpha_{i,t})\, g, \tag{3}$$

where $g > 0$ represents the highest level of a country's GHG emissions, and $0 \leq \alpha_{i,t} \leq 1$ applies. Country $i$ can either completely avoid emissions ($\alpha_{i,t} = 1$), emit the highest level ($\alpha_{i,t} = 0$) or emit something in between. Reducing emissions is costly, and we assume that a country's abatement costs increase with the level of emission reduction:

$$C_{i,t} = \alpha_{i,t} c, \tag{4}$$

where the parameter $c > 0$ represents the costs to a country of abating $g$.

One country's emission of $g_{i,t}$ units of GHGs in period $t$ causes additional climate costs for every country in the amount of

$$\Delta w_t = \sum_{\tau=t}^{\infty} (q\delta)^{\tau-t} g_{i,t} = \frac{(1-\alpha_{i,t})\, g}{1-q\delta}, \tag{5}$$

where $0 < \delta < 1$ represents the countries' common discount factor.

In the following, we assume that each country's costs of abating $g$ in period $t$ exceed the climate costs to the country that are induced by the emission of $g$ in period $t$:

(A1) $\qquad c > \dfrac{g}{1-q\delta}.$

Each single country does not internalize the climate costs imposed on all other countries but solely considers its own climate costs when determining the level of GHG emissions. Since it is more costly to reduce emissions than to handle climate change, no country will rationally reduce emissions on its own. Therefore, the Nash-equilibrium is characterized by $\alpha^* = 0$.

## 3 Climate negotiations

Now we consider an indefinite game of climate negotiations. We call the equilibrium with zero emissions climate agreement. Payoffs of a climate agreement are as follows: Let $k$ be the number of countries that participate in the agreement and reduce emissions to zero. Each country saves climate costs in the amount of

$$\Delta w_t k = \frac{g}{1-q\delta} k. \tag{6}$$

If no country reduces emissions, each country's payoff equals 0. Those countries that reduce emissions to zero have to bear abatement costs in the amount of $c$. Their payoff equals

$$\frac{g}{1-q\delta} k - c. \tag{7}$$

We assume for the number of negotiating countries $N$ that

(A2) $\qquad N > \left\lfloor \dfrac{c(1-q\delta)}{g} \right\rfloor,$

where the brackets symbolize the floor function which refers to the largest previous integer. From assumption (A2) follows that a global climate agreement with full participation is a Pareto improvement to the Nash equilibrium because

$$\frac{g}{1-q\delta}N - c > 0. \tag{8}$$

## 4 Punishment

Similar to Froyn and Hovi (2008), a global climate agreement with complete emission reduction can be achieved if a deviating country is punished by a restricted number of countries that emit GHGs in the following period, a strategy called penance-m: Every country reduces emissions, as long as all other countries reduce emissions, too. If a country deviates by emitting more than zero, it is punished by $m \in \mathbb{N}$ countries that emit $(1-\alpha)g$ with $0 \leq \alpha < 1$ in the following period while the $N - m - 1$ countries and the deviating country emit zero. If one of the $m$ punishing countries deviates from the punishment and emits more than $(1-\alpha)g$, it is punished by $m$ countries that will emit $(1-\alpha)g$ in the following period while the $N-m-1$ countries and the deviating country emit zero.

**Proposition 1.** *If assumptions (A1) and (A2) hold and abatement costs are not too high, i.e.*

$$c < \frac{2g}{1-q\delta},$$

*there exists a subgame perfect and weakly renegotiation-proof equilibrium with $m = 1$ and*

$$\alpha = 1 - \frac{c(1-q\delta) - g}{g\delta},$$

*if the weight that countries place on future payoffs is sufficiently high, i.e.*

$$\delta > \hat{\delta} = \frac{c-g}{g+cq}.$$

**Proof**

Subgame perfection in period $t$ requires that no country has an incentive to deviate from the treaty given any history $\tau = \ldots, t-2, t-1$. The payoff-maximizing deviation from an emission reduction obligation is to emit as much as possible. Therefore, if the following three conditions hold, the equilibrium is subgame perfect:

1. Given that all countries reduce emissions, a single country must have no incentive to emit $g$.

2. A country that has to punish and therefore to emit $(1-\alpha)g$ in period $t$ must have no incentive to emit $g$.

3. In a period of punishment, all $N-m$ non-punishing countries must have no incentive to emit $g$.

We assume that a country that has to punish in period $t$ does not belong to the $m$ punishing countries in period $t+1$. Condition 2 requires that

$$\delta^t \left( \frac{g}{1-q\delta}(N-m) + (m-1)\frac{\alpha g}{1-q\delta} \right) + \delta^{t+1} \left( \frac{g}{1-q\delta}(N-m) + m\frac{\alpha g}{1-q\delta} - c \right) \leq \delta^t \left( \frac{g}{1-q\delta}(N-m) + m\frac{\alpha g}{1-q\delta} - \alpha c \right) + \delta^{t+1} \left( \frac{g}{1-q\delta}N - c \right). \tag{9}$$

Hence, condition 2 is fulfilled if

$$m \geq \frac{\alpha}{1-\alpha}\frac{c(1-q\delta)-g}{g\delta}. \tag{10}$$

Condition 1 requires that

$$\delta^t \frac{g}{1-q\delta}(N-1) + \delta^{t+1} \left( \frac{g}{1-q\delta}(N-m) + m\frac{\alpha g}{1-q\delta} - c \right) \leq \delta^t \left( \frac{g}{1-q\delta}N - c \right) + \delta^{t+1} \left( \frac{g}{1-q\delta}N - c \right). \tag{11}$$

Solving for the number of punishing countries leads to

$$m \geq \frac{1}{(1-\alpha)}\frac{c(1-q\delta)-g}{g\delta}, \tag{12}$$

which means that if condition 1 is fulfilled, condition 2 is also fulfilled.

Condition 3 requires that

$$\delta^t \left( \frac{g}{1-q\delta}(N-m-1) + m\frac{\alpha g}{1-q\delta} \right) + \delta^{t+1} \left( \frac{g}{1-q\delta}(N-m) + m\frac{\alpha g}{1-q\delta} - c \right) \leq \delta^t \left( \frac{g}{1-q\delta}(N-m) + m\frac{\alpha g}{1-q\delta} - c \right) + \delta^{t+1} \left( \frac{g}{1-q\delta}N - c \right).$$

$$(13)$$

Solving for the number of punishing countries leads to

$$m \geq \frac{1}{(1-\alpha)} \frac{c(1-q\delta)-g}{g\delta}, \tag{14}$$

which is equivalent to the constraint derived from condition 1.

The agreement must also be renegotiation-proof (Farrell and Maskin, 1989: 330–331), i.e., in a period of punishment all $m$ punishing countries must have no incentive to reduce collectively emissions to zero,

$$\begin{aligned} \delta^t \left( \frac{g}{1-q\delta} N - c \right) + \delta^{t+1} \left( \frac{g}{1-q\delta} N - c \right) \leq \\ \delta^t \left( \frac{g}{1-q\delta} (N-m) + m \frac{\alpha g}{1-q\delta} - \alpha c \right) + \delta^{t+1} \left( \frac{g}{1-q\delta} N - c \right). \end{aligned} \tag{15}$$

Then,

$$m \leq \frac{c(1-q\delta)}{g} \tag{16}$$

must hold.

To summarize, if there is a natural number $m$ that fulfills

$$\frac{1}{(1-\alpha)} \frac{c(1-q\delta)-g}{g\delta} \leq m \leq \frac{c(1-q\delta)}{g},. \tag{17}$$

the climate agreement is subgame perfect and renegotiation-proof.

Let $c < 2g/(1-q\delta)$. Then, there is only one punishing country, $m = 1$. It follows that the fraction of emission reduction in a period of punishment equals

$$\alpha_{m=1} = 1 - \frac{c(1-q\delta)-g}{g\delta}. \tag{18}$$

Because $\delta \geq \hat{\delta}$, it holds that $\alpha_{m=1} \geq 0$.

By assumption (A1), $c(1-q\delta)/g > 1$, $\lfloor c(1-q\delta)/g \rfloor \geq 1$ and therefore $\alpha < 1$. Because $0 < q < 1$ and (A1), $c > g$ and therefore $\hat{\delta} > 0$.

Furthermore, because $c < \frac{2g}{1-q}$, it holds that $\hat{\delta} = (c-g)/(g+cq) < 1$.

□

**Corollary 1.** *For all* $0 \leq \alpha < 1$, $0 < g$, $0 < q < 1$ *and* $0 < \delta < 1$ *there exists a subgame perfect and weakly renegotiation-proof equilibrium with* $m \in \mathbb{N}$ *punishing countries if (A1), (A2) and (17) hold.*

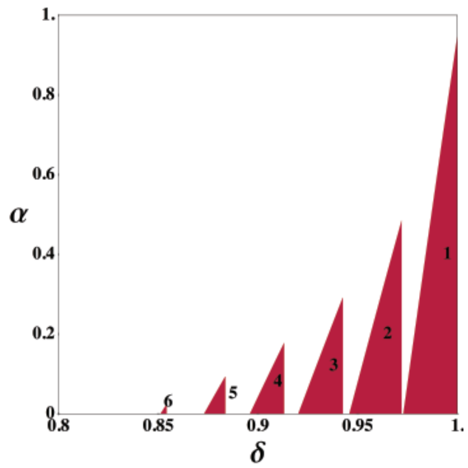


Figure 1: Areas of equilibria with $c = 35, g = 1, N = 150$ and $q = 0.97$.

Figure 1 illustrates equilibrium areas given by corollary 1 as a combination of the fraction of emission reduction for punishment $\alpha$ and the discount factor $\delta$ for various numbers of punishing countries. For every given number of punishing countries $m \in \mathbb{N}$, the discount factor must be high enough in order to fulfill the three conditions for subgame perfection. Furthermore, it is capped by the condition for renegotiation-proofness. Equilibrium areas for various discount factors are disjoint: There exists at most exactly one natural number of punishing countries for a certain discount factor that fulfills (17), because provided that assumption (A1) holds,

$$\frac{c(1-q\delta)}{g} - \frac{1}{(1-\alpha)}\frac{c(1-q\delta)-g}{g\delta} < 1. \tag{19}$$

Equilibrium areas are discontinuous in the discount factor. The discontinuity follows from the requirement that the number $m$ that fulfills condition (17) for weakly renegotiation-proofness has to be a natural number. For some combinations of $\alpha$ and $\delta$ only a non-natural number fulfills (17).

As Figure 1 shows, for $\delta < 0.85$ there is no combination of $m$ and $\alpha$ that leads to an equilibrium. Furthermore, a climate agreement is the more likely, the lower the fraction $\alpha$ of emission reduction for punishment. A hard punishment, i.e. full emission with $\alpha = 0$, enables a treaty for most discount factors. However, if there would be a treaty with a higher $\alpha$, the higher $\alpha$ leads to lower emissions during a punishment phase and is therefore

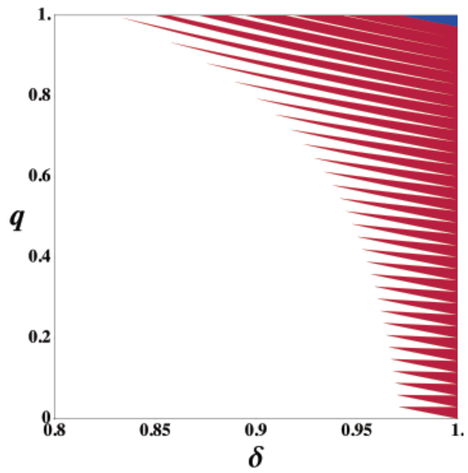


Figure 2: Areas of equilibria with $c = 35, g = 1, N = 150$ and $\alpha = 0$.

preferable from an environmental point of view.

Figure 2 illustrates equilibrium areas as a combination of the discount factor and the fraction of emissions that remain in the atmosphere per period when $\alpha = 0$. In the blue area there is no need for a treaty because (A1) does not hold. Successively below are the areas for $m = 1$ up to $m = 34$. The figure shows that the lower $q$, the more weight countries have to place on future payoffs if an agreement shall be reached. This is due to the fact that greenhouse gases with a long lifetime induce costs in all future periods as long as these gases are not dissipated, whereas short-lived greenhouse gases only induce costs for fewer periods. Furthermore, the number of countries needed in the punishment phase rises when $q$ shrinks.

## 5 Concluding Remarks

Greenhouse gases build up a stock that influences the climate. By explicitly modeling the depreciation of emitted gases we generalize some results from models that analyze a flow of emissions. Furthermore, we identify some effects that do not occur if emissions are modeled as a flow: Treaties are easier to reach for long lasting greenhouse gases than for short living gases. Because only a natural number of countries can punish, the areas where equilibria exist are not connected: There are parameter combinations where a treaty is possible with a low $\delta$ and a larger number of punishing countries. However, if $\delta$ is higher an equilibrium may not be possible due to the discontinuity in the discount factor. To treat short-lived gases more countries have to emit if a punishment is necessary.

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